DENSE FREE SUBGROUPS OF AUTOMORPHISM GROUPS OF HOMOGENEOUS PARTIALLY ORDERED SETS

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Let $1 \leq n \leq \omega$. Let A_n be a set of natural numbers less than n. Define $<$ on A_n so that for no $x, y \in A_n$ is $x < y$. Let $B_n = A_n \times \mathbb{Q}$ where $\mathbb{Q}$ is the set of rational numbers. Define $<$ on B_n so that $(k, p) < (m, q)$ iff $k = m$ and $p < q$. Let $C_n = B_n$ and define $<$ on C_n so that $(k, p) < (m, q)$ iff $p < q$. Finally, let $(D, <)$ be the universal countable homogeneous partially ordered set, that is a Fraïssé limit of all finite partial orders.

A structure is called ultrahomogeneous, if every embedding of its finitely generated substructure can be extended to an automorphism. Schmerl in [1] showed that there are only countably many, up to isomorphism, ultrahomogeneous countable partially ordered sets. More precisely he proved the following characterization.

Theorem 1. *Let $(H, <)$ be a countable partially ordered set. Then $(H, <)$ is ultrahomogeneous iff it is isomorphic to one of the following:*

(a) $(A_n, <)$ *for* $1 \leq n \leq \omega$*;*
(b) $(B_n, <)$ *for* $1 \leq n \leq \omega$*;*
(c) $(C_n, <)$ *for* $2 \leq n \leq \omega$*;*
(d) $(D, <)$.

Moreover, no two of the partially ordered sets listed above are isomorphic.

Consider automorphisms groups $\mathrm{Aut}(A_\omega) = S_\infty$, $\mathrm{Aut}(B_n)$, $\mathrm{Aut}(C_n)$ and $\mathrm{Aut}(D)$. We prove that each of these groups contains two elements f, g such that the subgroup generated by f and g is free and dense. By Schmerl's Theorem the automorphism group of a countable infinite partially ordered set is freely topologically 2-generated.

References

[1] J.H. Schmerl, Countable homogeneous partially ordered sets. Algebra Universalis 9 (1979), no. 3, 317–321.

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